

Storage and retrieval of continuous-variable polarization-entangled cluster states in atomic ensembles

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We present a proposal for storing and retrieving a continuous-variable quadripartite polarization-entangled cluster state, using macroscopic atomic ensembles in a magnetic field. The Larmor precession of the atomic spins leads to a symmetry between the atomic canonical operators. In this scheme, each of the four spatially separated pulses passes twice through the respective ensemble in order to map the polarization-entangled cluster state onto the long-lived atomic ensembles. The stored state can then be retrieved by another four read-out pulses, each crossing the respective ensemble twice. By calculating the variances, we analyzed the fidelities of the storage and retrieval, and our scheme is feasible under realistic experimental conditions.

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I. INTRODUCTION

In recent years, the investigation of continuous-variable (CV) quantum information has attracted much interest due to the relative simplicity and high efficiency in the generation, manipulation, and detection of the CV quantum state [1, 2]. As important resources, the CV entangled states have been widely applied in various quantum information processes, such as quantum teleportation [3], dense coding [4], entanglement swapping [5], quantum telecloning [6], and quantum computation [7, 8]. Quite recently, Zhang and Braunstein introduced CV cluster states [9], which are different from CV Greenberger-Horne-Zeilinger (GHZ) states, and the entanglement of the states is harder to destroy than that of GHZ states. In one-way quantum computation, CV cluster states play an important role [8], and all multimode Gaussian operations, performed through the cluster states, need only homodyne detection [10].

In view of the important functions of CV cluster states in quantum information processing, as well as the storage and retrieval of quantum states, we presented a proposal for the quantum memory of discrete-variable cluster states [11]. Furthermore, it is also worth initiating a study on the reversible memory of CV cluster states. In addition, compared to CV quadrature entanglement (corresponding to the amplitude and phase quadratures), the CV polarization entanglement (corresponding to the polarization basis) introduced by Korolkova *et al.* [12] has the advantages of compatibility with the spin variables of an atomic system [13] and of direct detection of its Stokes operators (not requiring complicated local oscillator measurements) [14–16]. Therefore, it is necessary to investigate the storage and retrieval of CV polarization-entangled cluster states (PECSs), and this paper is also an extension of our memory scheme for discrete-variable cluster states [11].

In this paper, we propose a protocol for efficiently storing and retrieving the CV PECSs in atomic ensembles. For quantum memory, one distinct method is based on the Faraday effect [17]. Using this approach, only partial transfer of the quantum information can be realized based on the quantum nondemolition (QND) interaction, but this shortcoming can

be overcome by additional measurement and feedback [18] or by use of a multipassage geometric configuration [19–21]. The most efficient schemes for this approach are eight passages of a single pulse [20] or two pulses each passing twice [21]. To reduce the complexity, Muschik *et al.* recently proposed a simple scenario [22] which can realize the complete transfer of a quantum state between light and an atomic ensemble in a magnetic field. Here, our scheme is also partly an extension of the existing work [22]. In our scheme, the storage of the quadripartite CV PECS can be achieved by using four spatially separated pulses (write-in pulses), which are initially in the CV PECS. Based on the Larmor precession of atoms in a magnetic field, each of the pulses passes through the corresponding atomic ensemble twice. After the storage time, the read-out pulses (another four pulses) are sent through the ensembles twice just like the write-in pulses. In this step, the CV PECS that is stored in the atomic ensembles will be transferred to the read-out pulses. Thus, we realize the storage and retrieval of the CV PECS, which approaches perfection with increasing coupling strength of the light fields interacting with the atomic ensembles. Moreover, we discuss the feasibility of our scheme, which can be realized under realistic experimental conditions.

This paper is organized as follows. In Sec. II, we propose a concrete scheme for storing and retrieving the CV PECS in atomic ensembles. In Sec. III, we analyze the fidelities of the storage and retrieval by calculating the variances. In Sec. IV, we discuss the experimental feasibility of the current scheme. Finally, in Sec. V, a conclusion is given.

II. STORAGE AND RETRIEVAL OF THE CV PECS

CV cluster-type (graph-type) states are defined as states that become zero eigenstates of a set of quadrature combinations in the limit of infinite squeezing [9, 23],

$$\left(\hat{p}_a - \sum_{b \in N_a} \hat{x}_b\right) \rightarrow 0, \quad \forall a \in G. \quad (1)$$

Here, the dimensionless amplitude and phase operators \hat{x}_a and \hat{p}_a , which satisfy the commutation relation $[\hat{x}_a, \hat{p}_a] = i/2$, correspond to the quadratures of an optical mode a with annihilation operator $\hat{a}_a = \hat{x}_a + i\hat{p}_a$. Each mode $a \in G$ corresponds to a vertex of the graph G while the modes $b \in N_a$ are the nearest neighbors of mode a . For the quadripartite CV cluster states, there are three different kinds of structures, i.e., the linear cluster state, the square cluster state, and the T-shaped cluster state [23, 24]. Here we use the linear cluster state as an example to describe the memory of the CV PECS whose corresponding operators can be expressed as [24]

$$\begin{aligned}
 \hat{X}_{L_1} &= \frac{1}{\sqrt{2}}e^{r_1}\hat{X}_{L_1}^{(0)} + \frac{1}{\sqrt{10}}e^{r_2}\hat{X}_{L_2}^{(0)} - \frac{2}{\sqrt{10}}e^{-r_3}\hat{P}_{L_3}^{(0)}, \\
 \hat{P}_{L_1} &= \frac{1}{\sqrt{2}}e^{-r_1}\hat{P}_{L_1}^{(0)} + \frac{1}{\sqrt{10}}e^{-r_2}\hat{P}_{L_2}^{(0)} + \frac{2}{\sqrt{10}}e^{r_3}\hat{X}_{L_3}^{(0)}, \\
 \hat{X}_{L_2} &= -\frac{1}{\sqrt{2}}e^{-r_1}\hat{P}_{L_1}^{(0)} + \frac{1}{\sqrt{10}}e^{-r_2}\hat{P}_{L_2}^{(0)} + \frac{2}{\sqrt{10}}e^{r_3}\hat{X}_{L_3}^{(0)}, \\
 \hat{P}_{L_2} &= \frac{1}{\sqrt{2}}e^{r_1}\hat{X}_{L_1}^{(0)} - \frac{1}{\sqrt{10}}e^{r_2}\hat{X}_{L_2}^{(0)} + \frac{2}{\sqrt{10}}e^{-r_3}\hat{P}_{L_3}^{(0)}, \\
 \hat{X}_{L_3} &= -\frac{2}{\sqrt{10}}e^{r_2}\hat{X}_{L_2}^{(0)} - \frac{1}{\sqrt{10}}e^{-r_3}\hat{P}_{L_3}^{(0)} - \frac{1}{\sqrt{2}}e^{-r_4}\hat{P}_{L_4}^{(0)}, \\
 \hat{P}_{L_3} &= -\frac{2}{\sqrt{10}}e^{-r_2}\hat{P}_{L_2}^{(0)} + \frac{1}{\sqrt{10}}e^{r_3}\hat{X}_{L_3}^{(0)} + \frac{1}{\sqrt{2}}e^{r_4}\hat{X}_{L_4}^{(0)}, \\
 \hat{X}_{L_4} &= \frac{2}{\sqrt{10}}e^{-r_2}\hat{P}_{L_2}^{(0)} - \frac{1}{\sqrt{10}}e^{r_3}\hat{X}_{L_3}^{(0)} + \frac{1}{\sqrt{2}}e^{r_4}\hat{X}_{L_4}^{(0)}, \\
 \hat{P}_{L_4} &= -\frac{2}{\sqrt{10}}e^{r_2}\hat{X}_{L_2}^{(0)} - \frac{1}{\sqrt{10}}e^{-r_3}\hat{P}_{L_3}^{(0)} + \frac{1}{\sqrt{2}}e^{-r_4}\hat{P}_{L_4}^{(0)}.
 \end{aligned} \tag{2}$$

In this paper, the operators \hat{X}_{L_i} and \hat{P}_{L_i} are the light polarization canonical variables where the superscript (0) denotes the initial condition and $r_i (i = 1, 2, 3, 4)$ is the polarization squeezing parameter of the i th “mode”.

The schematic diagram of the proposed experimental system is shown in Fig. 1 where four atomic ensembles spin polarized along x are placed in the magnetic field and used as memory cells. The relevant level structure of the atoms is shown in Fig. 2. These four ensembles form four equivalent channels, each of which is used to store the corresponding polarization-squeezed light field. Using a Holstein-Primakoff transformation and approximation, the atomic canonical operators \hat{X}_{A_i} and \hat{P}_{A_i} (the subscript A denotes the atomic ensemble) can be defined as the y and z components of the collective angular momentum \hat{J} , i.e., $\hat{X}_{A_i} = (\hat{J}_y)_i / \sqrt{\langle (\hat{J}_x)_i \rangle}$ and $\hat{P}_{A_i} = (\hat{J}_z)_i / \sqrt{\langle (\hat{J}_x)_i \rangle}$ where $i = 1, 2, 3, 4$. The Larmor precession of the atomic spins in the magnetic field is a characteristic feature of this protocol; it leads to a symmetry between the operators \hat{X}_A and \hat{P}_A . Four spatially separated optical pulses propagate along z where each pulse consists of a strong x -polarized component and a copropagating quantum field with y polarization. The classical light field drives the $m = \pm 1/2 \rightarrow m' = \pm 1/2$ transitions while the copropagating quantum field couples to $m = \mp 1/2 \rightarrow m' = \pm 1/2$. For input light fields, the Stokes vector component $(\hat{S}_x)_i$ is

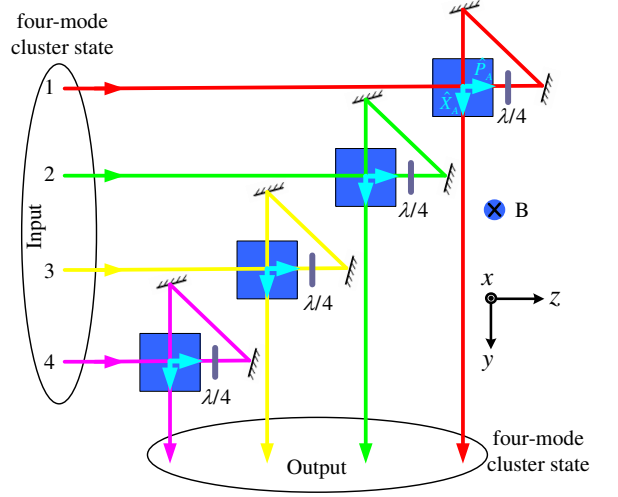


FIG. 1: (Color online) Schematic for storage or retrieval of quadripartite CV cluster states. Here, four optical pulses (four-mode cluster state) each propagates through the corresponding x -polarized atomic spin ensemble twice. For each pulse, the \hat{V}_1 interaction happens during the first passage (z direction), then each pulse is sent through a $\lambda/4$ wave plate, and is reflected back onto the atomic ensemble, thus the \hat{V}_2 interaction occurs during the second passage (y direction).

a macroscopic classical quantity and we can define the operators $\hat{X}_{L_i} = (\hat{S}_y)_i / \sqrt{\langle (\hat{S}_x)_i \rangle}$ and $\hat{P}_{L_i} = (\hat{S}_z)_i / \sqrt{\langle (\hat{S}_x)_i \rangle}$ where $i = 1, 2, 3, 4$. According to Eq. (1), the four light pulses are the P -squeezed states, i.e., the variance of the \hat{S}_z Stokes operator is less than the coherent state value [12].

In order to explain our scheme better, we first consider one pulse of four spatially separated pulses. When a pulse passes through the cubic atomic ensemble twice as depicted in Fig. 1, the Hamiltonian of the interaction can be written as [22, 25]

$$\begin{aligned}
 \hat{H} &= \hat{H}_A + \hat{H}_L + \hat{V}_1 + \hat{V}_2, \\
 \hat{V}_1 &= \frac{\hbar\kappa}{\sqrt{T}}\hat{P}_A\hat{P}_L(0), \\
 \hat{V}_2 &= \frac{\hbar\kappa}{\sqrt{T}}\hat{X}_A\hat{X}_L(d),
 \end{aligned} \tag{3}$$

where \hat{H}_A represents the Zeeman splitting of the atomic ground state, causing Larmor precession of the transverse spin components \hat{X}_A and \hat{P}_A , and \hat{H}_L describes the free propagation of light. The two left terms \hat{V}_1 and \hat{V}_2 denote the off-resonant scattering interaction in the first and the second passage of the pulse respectively, κ is the coupling strength, and T is the duration of the pulse. The argument “0” of the light operator \hat{P}_L in \hat{V}_1 indicates that the first scattering interaction occurs at $r = 0$. The argument “ d ” of the light operator \hat{X}_L in \hat{V}_2 means that the second scattering interaction happens after the light has traveled some distance d in the small loop between the mirrors. After the first passage, the pulse is sent through a quarter-wave plate, which interchanges the light operators \hat{P}_L and \hat{X}_L . Furthermore, the changed geometry also interchanges the atomic operators \hat{P}_A and \hat{X}_A .

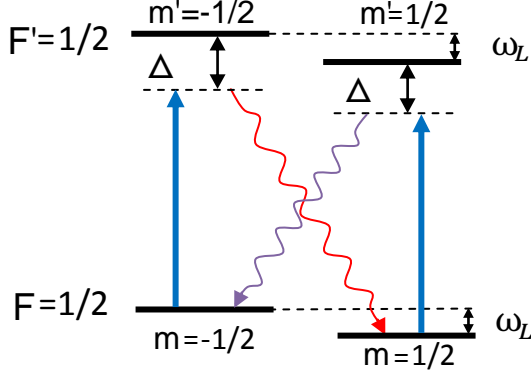


FIG. 2: (Color online) Level structure of the atoms. The classical field (thick lines) drives the $m = \pm 1/2 \rightarrow m' = \pm 1/2$ transitions, and the co-propagating quantum fields (wavy lines) couples to $m = \pm 1/2 \rightarrow m' = \pm 1/2$. ω_L is the Larmor frequency, and Δ is the detuning.

Now, we explain the storage and retrieval of the four-mode CV PECS. For the procedure of storage, after the four pulses pass twice through the four corresponding atomic ensembles (see Fig. 1), whose operators \hat{X}_{Li}^{in} and \hat{P}_{Li}^{in} ($i = 1, 2, 3, 4$) have the forms of Eq. (2), the output of the atomic ensembles will have the following relations with the input of the atomic ensembles and the light pulses [22]:

$$\begin{pmatrix} \hat{X}_{A_1}^{out} \\ \hat{P}_{A_1}^{out} \\ \hat{X}_{A_2}^{out} \\ \hat{P}_{A_2}^{out} \\ \hat{X}_{A_3}^{out} \\ \hat{P}_{A_3}^{out} \\ \hat{X}_{A_4}^{out} \\ \hat{P}_{A_4}^{out} \end{pmatrix} = e^{-\kappa^2/2} \begin{pmatrix} \hat{X}_{A_1}^{in} \\ \hat{P}_{A_1}^{in} \\ \hat{X}_{A_2}^{in} \\ \hat{P}_{A_2}^{in} \\ \hat{X}_{A_3}^{in} \\ \hat{P}_{A_3}^{in} \\ \hat{X}_{A_4}^{in} \\ \hat{P}_{A_4}^{in} \end{pmatrix} + \sqrt{1 - e^{-\kappa^2}} \begin{pmatrix} \hat{X}_{L_1}^{in} \\ \hat{P}_{L_1}^{in} \\ \hat{X}_{L_2}^{in} \\ \hat{P}_{L_2}^{in} \\ \hat{X}_{L_3}^{in} \\ \hat{P}_{L_3}^{in} \\ \hat{X}_{L_4}^{in} \\ \hat{P}_{L_4}^{in} \end{pmatrix}. \quad (4)$$

Obviously, for large values of κ , $\hat{X}_{Li}^{in} \rightarrow \hat{X}_{Li}^{out}$ and $\hat{P}_{Li}^{in} \rightarrow \hat{P}_{Li}^{out}$ ($i = 1, 2, 3, 4$); thus, the output canonical operators of atomic ensembles satisfy the following correlations:

$$\begin{aligned} \hat{P}_{A_1}^{out} - \hat{X}_{A_2}^{out} &= \sqrt{2}e^{-r_1}\hat{P}_{A_1}^{(0)}, \\ \hat{P}_{A_2}^{out} - \hat{X}_{A_1}^{out} - \hat{X}_{A_3}^{out} &= \frac{\sqrt{10}}{2}e^{-r_3}\hat{P}_{A_3}^{(0)} + \frac{1}{\sqrt{2}}e^{-r_4}\hat{P}_{A_4}^{(0)}, \\ \hat{P}_{A_3}^{out} - \hat{X}_{A_2}^{out} - \hat{X}_{A_4}^{out} &= \frac{1}{\sqrt{2}}e^{-r_1}\hat{P}_{A_1}^{(0)} - \frac{\sqrt{10}}{2}e^{-r_2}\hat{P}_{A_2}^{(0)}, \\ \hat{P}_{A_4}^{out} - \hat{X}_{A_3}^{out} &= \sqrt{2}e^{-r_4}\hat{P}_{A_4}^{(0)}. \end{aligned} \quad (5)$$

Thus, the storage is realized perfectly.

For the procedure of retrieval, another four read-out pulses are sent through the four corresponding atomic ensembles twice with the same interaction happening. Considering the contribution of the atomic ensembles and the read-out pulses,

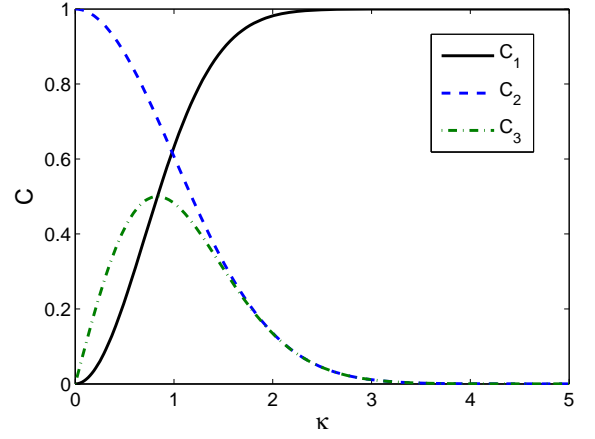


FIG. 3: (Color online) The coefficients versus coupling strength κ . C_1 , C_2 and C_3 are the coefficients of Eq. (7).

the output can be written as [22]

$$\begin{pmatrix} \hat{X}_{L_1}^{'out} \\ \hat{P}_{L_1}^{'out} \\ \hat{X}_{L_2}^{'out} \\ \hat{P}_{L_2}^{'out} \\ \hat{X}_{L_3}^{'out} \\ \hat{P}_{L_3}^{'out} \\ \hat{X}_{L_4}^{'out} \\ \hat{P}_{L_4}^{'out} \end{pmatrix} = -\sqrt{1 - e^{-\kappa^2}} \begin{pmatrix} \hat{X}_{A_1}^{in} \\ \hat{P}_{A_1}^{in} \\ \hat{X}_{A_2}^{in} \\ \hat{P}_{A_2}^{in} \\ \hat{X}_{A_3}^{in} \\ \hat{P}_{A_3}^{in} \\ \hat{X}_{A_4}^{in} \\ \hat{P}_{A_4}^{in} \end{pmatrix} + e^{-\kappa^2/2} \begin{pmatrix} \hat{X}_{L_1}^{in} \\ \hat{P}_{L_1}^{in} \\ \hat{X}_{L_2}^{in} \\ \hat{P}_{L_2}^{in} \\ \hat{X}_{L_3}^{in} \\ \hat{P}_{L_3}^{in} \\ \hat{X}_{L_4}^{in} \\ \hat{P}_{L_4}^{in} \end{pmatrix}, \quad (6)$$

where the prime is used to distinguish the retrieval process from the storage process.

After the whole storage and subsequent retrieval procedure we consider the final output. For atomic ensembles, the output operators of the write-in procedure are the input variables of the read-out procedure, i.e., $\hat{X}_{Ai}^{out} = \hat{X}_{Ai}^{'in}$ and $\hat{P}_{Ai}^{out} = \hat{P}_{Ai}^{'in}$. By inserting Eq. (4) into Eq. (6), the final output of lights can be written as

$$\begin{pmatrix} \hat{X}_{L_i}^{'out} \\ \hat{P}_{L_i}^{'out} \end{pmatrix} = -C_1 \begin{pmatrix} \hat{X}_{Ai}^{in} \\ \hat{P}_{Ai}^{in} \end{pmatrix} + C_2 \begin{pmatrix} \hat{X}_{L_i}^{in} \\ \hat{P}_{L_i}^{in} \end{pmatrix} - C_3 \begin{pmatrix} \hat{X}_{Ai}^{in} \\ \hat{P}_{Ai}^{in} \end{pmatrix}, \quad (7)$$

where $C_1 = 1 - e^{-\kappa^2}$, $C_2 = e^{-\kappa^2/2}$, and $C_3 = e^{-\kappa^2/2}\sqrt{1 - e^{-\kappa^2}}$. In Fig. 3, these coefficients are plotted versus the coupling strength κ . From the diagram, it is obvious that with increasing κ the coefficient C_1 increases, C_2 decreases, and C_3 attains a maximum. When κ increases to a certain value, C_1 converges to 1, and C_2 and C_3 tend to zero. The final canonical operators of output for the read-out pulses will have the following relations with those of the input for the write-in pulses:

$$\hat{X}_{L_i}^{'out} = -\hat{X}_{L_i}^{in}, \quad \hat{P}_{L_i}^{'out} = -\hat{P}_{L_i}^{in}, \quad (8)$$

where the noise from the atomic ensembles (\hat{X}_{Ai}^{in} and \hat{P}_{Ai}^{in}) and the pulses of light ($\hat{X}_{L_i}^{in}$ and $\hat{P}_{L_i}^{in}$) will disappear. So the

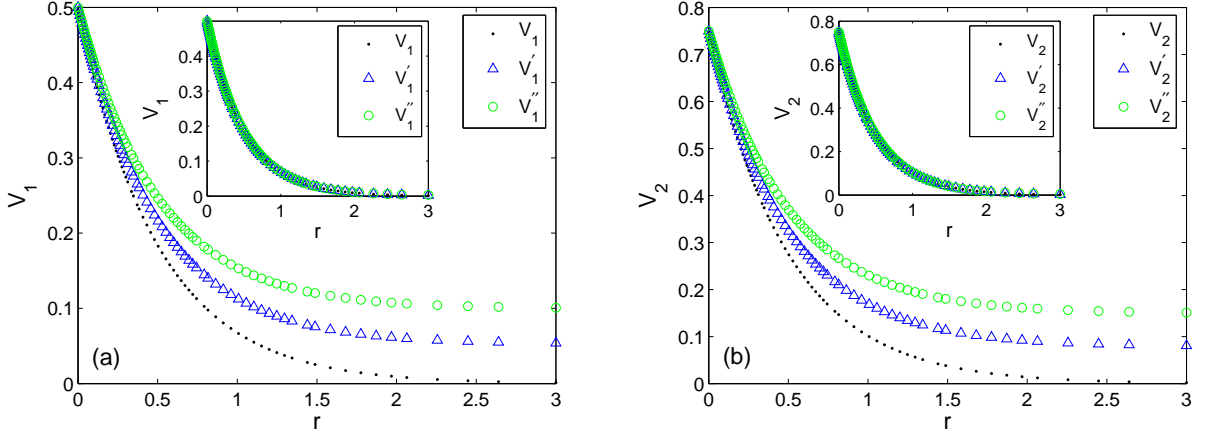


FIG. 4: (Color online) (a) The variances V_1 , V_1' and V_1'' versus the squeezing parameter r . (b) The variances V_2 , V_2' and V_2'' versus the squeezing parameter r . Here, the big graphs and the inserted small graphs correspond to $\kappa = 1.5$ and $\kappa = 2.5$, respectively.

final output state of the pulses also satisfies the definition of the CV PECS for large κ .

III. FIDELITY ANALYSIS

In this section, we will analyze the fidelities of the storage and the retrieval by calculating the variances. For the sake of simplicity, we assume the squeezing parameters r of the four modes of the CV PECS to be identical. With $V(\hat{X}_{L_i}^{(0)}) = V(\hat{P}_{L_i}^{(0)}) = 1/4$, $V(\hat{X}_{A_i}^{in}) = V(\hat{P}_{A_i}^{in}) = 1/4$, and $V(\hat{X}_{L_i}'^{in}) = V(\hat{P}_{L_i}'^{in}) = 1/4$, we can give the expressions of the variances, which can be used to characterize the entanglement [26, 27]. For the CV PECS of light to be stored in the atomic ensembles, the variances of the linear combinations of the components are

$$\begin{aligned} V_1(\hat{P}_{L_1}^{in} - \hat{X}_{L_2}^{in}) &= V_4(\hat{P}_{L_4}^{in} - \hat{X}_{L_3}^{in}) = \frac{1}{2}e^{-2r}, \\ V_2(\hat{P}_{L_2}^{in} - \hat{X}_{L_1}^{in} - \hat{X}_{L_3}^{in}) &= V_3(\hat{P}_{L_3}^{in} - \hat{X}_{L_2}^{in} - \hat{X}_{L_4}^{in}) = \frac{3}{4}e^{-2r}. \end{aligned} \quad (9)$$

For the state stored in the atomic ensembles, the variances are

$$\begin{aligned} V_1'(\hat{P}_{A_1}^{out} - \hat{X}_{A_2}^{out}) &= V_4'(\hat{P}_{A_4}^{out} - \hat{X}_{A_3}^{out}) \\ &= \frac{1}{2}(1 - e^{-\kappa^2})e^{-2r} + \frac{1}{2}e^{-\kappa^2}, \\ V_2'(\hat{P}_{A_2}^{out} - \hat{X}_{A_1}^{out} - \hat{X}_{A_3}^{out}) &= V_3'(\hat{P}_{A_3}^{out} - \hat{X}_{A_2}^{out} - \hat{X}_{A_4}^{out}) \\ &= \frac{3}{4}(1 - e^{-\kappa^2})e^{-2r} + \frac{3}{4}e^{-\kappa^2}. \end{aligned} \quad (10)$$

In addition, for the final output state retrieved from the atomic ensembles, the variances have the following expressions:

$$\begin{aligned} V_1''(\hat{P}_{L_1}'^{out} - \hat{X}_{L_2}'^{out}) &= V_4''(\hat{P}_{L_4}'^{out} - \hat{X}_{L_3}'^{out}) \\ &= \frac{1}{2}(1 - e^{-\kappa^2})^2e^{-2r} + \frac{1}{2}(2 - e^{-\kappa^2})e^{-\kappa^2}, \end{aligned}$$

$$\begin{aligned} V_2''(\hat{P}_{L_2}'^{out} - \hat{X}_{L_1}'^{out} - \hat{X}_{L_3}'^{out}) &= V_3''(\hat{P}_{L_3}'^{out} - \hat{X}_{L_2}'^{out} - \hat{X}_{L_4}'^{out}) \\ &= \frac{3}{4}(1 - e^{-\kappa^2})^2e^{-2r} + \frac{3}{4}(2 - e^{-\kappa^2})e^{-\kappa^2}. \end{aligned} \quad (11)$$

Based on these equations, the plots of Fig. 4 analyze the fidelities of this scenario. They show that when $\kappa = 1.5$, the variances corresponding to Eq. (9) tend to zero in the limit of infinite squeezing (i.e., $r \rightarrow \infty$). However, the variance corresponding to Eq. (10) converges to a smaller constant whereas Eq. (11) converges to a larger constant when $r \rightarrow \infty$. Therefore, the variances increase gradually (i.e., the entanglement decreases gradually) for Eqs. (9), (10), and (11). Moreover, when $\kappa = 2.5$, we find that all the variances converge to zero in the limit of infinite squeezing, and they have almost the same evolution curves. So, the state stored in atomic ensembles and the state retrieved from atomic ensembles have the same entanglement properties with the initial CV PECS for large κ . Thus, the fidelities of the storage and subsequent retrieval are high for larger κ and low for smaller κ . With the increase of the coupling strength κ , the protocol will approach perfection, which can also be understood easily from Eqs. (4)-(7).

IV. DISCUSSION

Because the polarization entanglement has some advantages that the quadrature entanglement does not have, we investigate CV PECSs in our scheme. For quadrature entanglement, the CV cluster states can be generated deterministically through offline squeezing and passive linear optics [23]; moreover, some other proposals have also been put forward recently [24, 27, 28]. Currently, for polarization entanglement, schemes for preparing CV cluster states have not been proposed. However, there are two ways to generate CV polarization cluster states. CV polarization entanglement was first realized by Bowen *et al.* [15], who reported the

experimental transformation of the CV quadrature entanglement between two optical beams into CV polarization entanglement. Hence, people can also transform quadrature cluster states into polarization cluster states, taking advantage of this method. Very recently, another scheme for efficiently creating CV polarization-entangled states was experimentally demonstrated by Dong *et al.* [16], who used two polarization-squeezed input states to generate polarization entanglement directly. Thus, CV PECSs can also be generated based on the scheme of Dong *et al.*

Our protocol is based on the Larmor precession of the atomic spins in an external field by which the effect of damping is distributed among both atomic canonical operators [22]. This leads to a symmetry between the atomic canonical operators \hat{X}_A and \hat{P}_A , which is a crucial feature for this proposal. In addition, the fidelities of our protocol can be enhanced by increasing the coupling strength κ . The above analysis shows that when κ increases to 2.5, the results are almost perfect. In Ref. [29], a detailed analysis of the coupling strength κ is given where $\kappa \sim 5$ is obtainable by adjusting the detuning and the loss. Moreover, the authors of Refs. [19, 20, 30] have also discussed the coupling parameter κ , which can also be changed by adjusting other parameters in experiment. So our protocol does not place a very high demand on the value of the coupling strength κ and is experimentally feasible under realistic technological conditions.

V. CONCLUSION

In conclusion, we have proposed a protocol for storing and retrieving a CV PECS in macroscopic atomic ensembles. Tak-

ing the quadripartite linear CV cluster state as an example, we have investigated the realization of this scheme in detail and have discussed the fidelities by calculating the variances. In fact, for an arbitrary multipartite CV PECS, storage and retrieval can be achieved using this protocol. Our scheme is made perfectly by enhancing the coupling strength and is feasible within the current experimental technology.

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